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$AN = AM\cos\beta = r\cos\beta - a\sin\beta$.

Volume of $ADEG = 2\pi ar^2$.

Volume of $EGF = 2\int_0^{2r}\int_0^{\sqrt{(2rx-x^2)}}\int_0^{x\tan\beta} dxdydz = \pi r^3\tan\beta$.

[This result comes easily without the calculus].

Substituting in equation (1), transposing, dividing through by $\cos\beta$, rearranging, and clearing of fractions,

$$5r^2\tan^3\beta + 16ar\tan^2\beta + 2(8a^2 - 3r^2)\tan\beta - 16ar = 0.$$

Substituting in this, $\frac{1}{2}h - r\tan\beta$ for a, and reducing,

$$5r^2\tan^3\beta - 8rh\tan^2\beta + 2(5r^2 + 2h^2)\tan\beta - 8rh = 0,$$

from which β may be found.

AVERAGE AND PROBABILITY.

62. Proposed by O. S. KIBLER, Superintendent of Schools, Middleburg, O.

A bag contains any number of balls, which are equally likely to be white or black; one is drawn and found to be white. Show that the chance of drawing another white one, the first ball not being replaced, is two-thirds. [From *C. Smith's Treatise on Algebra*, page 615].

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Science, Chester High School, Chester, Pa.

If m balls are drawn and turn out white, the chance that n others drawn will be white is: $p = [(m+1)/(m+n+1)]$. In the problem, $m = n = 1$.

$\therefore p = \frac{2}{3}$. Otherwise, $p = \int_0^1 x^2dx / \int_0^1 xdx = \frac{2}{3}$.

This is the simplest case of the article on page 107, No. 4, Vol. II. of the MONTHLY.

64. Proposed by REV. W. A. WHITWORTH, A. M.

O is a given point within a triangle; P is a random point within the same. The line through O and P is produced so as to divide the triangle into a trapezium and a triangle. Find the average area of this triangle. [From the *Educational Times*, London, Eng.]

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Science, Chester High School, Chester, Pa.

Let $\triangle$ be the average area required, $\triangle_1$ the average area of BFG; (m, n) coördinates of O. Then the coördinates of E are $(b, bn/m)$; of D, $[b^2n/(ab - am + bn), abn/(ab - am + bn)]$.

Area $ABC = \frac{1}{2}ab\sin C$.

Area $ACO = \frac{1}{2}bn\sin C$.

Area $ADO = bn(am - bn)\sin C/[2(ab - am + bn)]$.

Area $COE = bn(b - m)\sin C/(2m)$.

Area $DOEB = ABC - (ACO + ADO + COE)$.

$\therefore$ Area $DOEB=(a^2b^2m-a^2bm^2+2ab^2mn-abm^2n+b^2n^2m$
$-ab^3n-b^3n^2)\sin C/[2m(ab-am+bn)]$.

The area $DOEB$ does not vary.

The average area of $GOE+DOF=\frac{1}{2}(COE+AOD)$

$$=\frac{(2abm^2n-2b^2mn^2-2ab^2mn+ab^3n+b^3n^2)\sin C}{4m(ab-am+bn)}.$$

$\therefore\ \triangle_1=DOEB+\frac{1}{2}COE+AOD$.

$$\therefore\ \triangle_1=\frac{(2a^2b^2m-2a^2bm^2+2ab^2mn-ab^3n-b^3n^2)\sin C}{4m(ab-am+bn)}.$$

Similarly, $\triangle_2=\dfrac{(2b^2c^2m-2b^2cm^2+2bc^2mn-bc^3n-c^3n^2)\sin C}{4m(cb-bm+cn)}$.

$$\triangle_3=\frac{(2a^2c^2m-2ac^2m^2+2a^2cmn-a^3cn-a^3n^2)\sin C}{4m(ac-cm+an)}.$$

$\triangle=\frac{1}{3}(\triangle_1+\triangle_2+\triangle_3)$. If $a=b=c$ and $m=2n=\frac{2}{3}a$, $\triangle=\frac{1}{8}(a^2\sqrt{3})=\frac{1}{2}$ area of the triangle.

DIOPHANTINE ANALYSIS.

65. **Proposed by MANSFIELD MERRIMAN, C. E., Ph. D., Professor of Civil Engineering, Lehigh University, South Bethlehem, Pa.**

Show that the number 1521 can be expressed in seven different ways as the sum of three perfect squares. Can more than seven different ways be found?

I. **Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.**

With the aid of a table that I devised for finding the sum of two squares equal to a number, I found 1521 expressed in *eight* different ways as the sum of three perfect squares. $1521=39^2=2^2+19^2+34^2=2^2+26^2+29^2=9^2+12^2+36^2=10^2+14^2+35^2=13^2+14^2+34^2=13^2+26^2+26^2=14^2+22^2+29^2=19^2+22^2+26^2$.

II. **Solution by CHARLES CARROLL CROSS, Libertytown, Md.**

The terminal figures of square numbers are, 0, 1, 4, 5, 6 and 9. These may be combined,—taking three at a time, so that the terminal figure will be one in the following manner: (5+5+1)..... (1); (9+1+1)......(2); (1+0+0)..... (3); (9+6+6)......(4); (6+4+1)..... (5); and (6+5+0)...... (6). It readily appears that it is impossible to combine any square numbers represented by (1), (2) and (3) so as to make 1521.

Those of (4) are found to be $13^2+34^2+14^2$ and $13^2+26^2+26^2$; of (5), $34^2+2^2+19^2$, $26^2+22^2+19^2$, $26^2+2^2+29^2$ and $14^2+22^2+29^2$; and of (6), $14^2+36^2+10^2$,—seven in all.